Applications of Electrochemistry – Faraday’s Law

Faraday’s Law: the amount of a substance produced or consumed in an electrolysis reaction is directly proportional to the quantity of electricity that flows through the circuit.

Electrical measurements:

Electric current – flow of electrons through an external current, $I$
  - measured in units of Ampere (A)

Electric charge – quantity of electricity, $Q$
  - the product of the current flowing through a circuit by the amount of time it flows
  - measured in units of Coulomb (C)

  Coulomb: the quantity of electricity that flows through a circuit in one second if the current is one ampere ie. $1 \text{A} = 1 \text{C/s}$

This relationship can also be expressed mathematically by:

$$\text{Charge (coulomb)} = \text{current (in ampere)} \times \text{time (in seconds)}$$

or

$$Q = It$$

Example:
Find the quantity of electricity that results from a current of 3.50 A flowing for 6.00 min.

$$Q = It$$
$$Q = 3.50 \text{A} \times 6.00 \text{min} \times (60 \text{s/1 min})$$
$$Q = 1.26 \times 10^3 \text{C}$$

Electrochemical Stoichiometry:

The equation for an electrode reaction relates the number of moles of substance consumed or produced during the reaction to the quantity of charge that passes through the cell

$$\text{Ag}^+_{(aq)} + 1\text{e}^- \rightarrow \text{Ag}_{(s)}$$

In order to produce one mole of solid silver one mole of electrons must be picked up by one mole of silver ions.

$$\text{Au}^{3+}_{(aq)} + 3\text{e}^- \rightarrow \text{Au}_{(s)}$$

To produce one mole of solid gold three moles of electrons must be picked up by one mole of gold ions.

- To perform stoichiometric calculations for electrochemical and electrolytic cells we also need to know the charge on one mole of electrons.
- The charge on one mole of electrons is one Faraday (1 F) or 96500 C
- Normally, the rounded number $96 \ 500 \text{C/mol}$ is used in calculations involving Faraday’s constant. Note that this number only has three significant figures.

Moles of electrons, $n_e = Q/F$ 
  ie. $n = \text{charge/charge per mole of electrons}$

- Since $Q = It$ then $n_e = (I \ t)/F$
  Ie. $n_e = \text{current(A) x time/charge per mole of electrons}$
To Calculate the Mass of an Electrolysis Product:

Steps:

1. Find the total moles of electrons that passed through the circuit using \( n_e = \frac{I \cdot t}{F} \) or \( n_e = \frac{Q}{F} \)
2. Find the total moles of metal produced using the half-reaction and mole ratio \( n_R = n_G \cdot \frac{R}{G} \)
3. Convert the moles of metal to mass using \( m = nM \)

Example 1:
Calculate the mass of zinc plated onto the cathode of an electrolytic cell by a current of 750 mA in 3.25 h.

\[
\text{Zn}^{2+}_{(aq)} + 2e^- \rightarrow \text{Zn}_{(s)}
\]

I = 750 mA = 0.750 A
\( t = 3.25 \text{ h} \times (60 \text{ min/1 h}) \times (60 \text{ s/1 min}) = 1.17 \times 10^4 \text{ s} \)
\( F = 96500 \text{ C/mol} \)

Step 1:
\( n_e = \frac{I \cdot t}{F} = \frac{(0.750 \text{ A} \times 1.17 \times 10^4 \text{ s})}{96500 \text{ C/mol}} = 0.0909 \text{ mol e}^- \) (this is the total moles of electrons)

Step 2:
\[
\text{Zn}^{2+}_{(aq)} + 2e^- \rightarrow \text{Zn}_{(s)}
\]
\( n_R = n_G \cdot \frac{R}{G} = 0.0909 \text{ mol e}^- \times \frac{1}{2} \)
\( n_{\text{Zn(s)}} = 0.0455 \text{ mol of Zn produced} \)

Step 3:
\( m = nM \)
\( m = 0.0455 \text{ mol} \times 65.39 \text{ g/mol} \)
\( m = 2.98 \text{ g} \)

Example 2
How many grams of gold will be deposited when 5000 C pass through a solution by the following equation.

\[
\text{AuCl}_4^{-}_{(aq)} + 3e^- \rightarrow \text{Au}_{(s)} + 4\text{Cl}^{-}_{(aq)}
\]

\( Q = 5000 \text{ C} \)
\( F = 96500 \text{ C/mol} \)

Step 1:
\( n_e = \frac{Q}{F} \)
\( n = \frac{5000 \text{ C}}{96500 \text{ C/mol}} = 0.0518 \text{ moles of electrons} \)

Step 2:
\[
\text{AuCl}_4^{-}_{(aq)} + 3e^- \rightarrow \text{Au}_{(s)} + 4\text{Cl}^{-}_{(aq)}
\]
\( n_R = n_G \cdot \frac{R}{G} = 0.0518 \text{ mol e}^- \times \frac{1}{3} \)
\( n_{\text{Au(s)}} = 0.0173 \text{ mol of Au(s)} \) will be produced

Step 3:
\( m = nM \)
\( m = 0.0173 \text{ mol} \times 196.97 \text{ g/mol} \)
\( m = 3.41 \text{ g} \)
Example 3:
Calculate the moles of Mg produced when a current of 60 amps is passed through a magnesium chloride solution for 4.00 h.

\[ \text{Mg}^{2+}_{(aq)} + 2e^- \rightarrow \text{Mg}_{(s)} \]

\[ I = 60 \text{ A} \quad F = 96,500 \text{ C/mol} \]
\[ t = 4.00 \text{ h} \]

Step 1: \( n_e = \frac{It}{F} \)
\[ n = \frac{(60 \text{ A})(1.44 \times 10^4 \text{ s})}{96,500 \text{ C/mol}} \]
\[ n = 8.95 \text{ mol of e}^- \]

Step 2:
\[ \begin{align*}
G & \quad R \\
\text{Mg}^{2+}_{(aq)} + 2e^- & \rightarrow \text{Mg}_{(s)} \\
\frac{n_R}{n} & = \frac{R}{G} \\
& = 8.95 \text{ mol e}^- \times \frac{1}{2} \\
n_{\text{Mg}_{(s)}} & = 4.48 \text{ mol of Mg produced}
\end{align*} \]

Step 3:
\[ m = nM \]
\[ m = 4.48 \text{ mol} \times 24.32 \text{ g/mol} \]
\[ m = 109 \text{ g} \]

Example 3:
How many hours would it take for a 3.50 A current to electroplate 129 g of gold according to the following equation.

\[ \text{Au(CN)}_2^{-}_{(aq)} + e^- \rightarrow \text{Au}_{(s)} + 2\text{CN}^{-}_{(aq)} \]

\[ I = 3.50 \text{ A} \]
\[ m = 129 \text{ g} \]
\[ F = 96,500 \text{ C/mol} \]
\[ t = ? \]

To solve for time, rearrange the equation \( n = \frac{It}{F} \) such that \( t = \frac{nF}{I} \) and convert mass to moles.

Step 1: Convert mass of gold to moles of gold
\[ n = \frac{m}{M} \]
\[ n = \frac{129}{196.97} \text{ g/mol} \]
\[ n = 0.655 \text{ mol of gold} \]

Step 2: Convert moles of gold to moles of electrons
\[ \text{Au(CN)}_2^{-}_{(aq)} + e^- \rightarrow \text{Au}_{(s)} + 2\text{CN}^{-}_{(aq)} \]
\[ \frac{n_R}{n} = \frac{R}{G} = 0.655 \text{ mol e}^- \times \frac{1}{1} \]
\[ n = 0.655 \text{ mol e}^- \]

Step 3: Determine the time
\[ t = \frac{n_eF}{I} \]
\[ t = \frac{0.655 \text{ mol} \times 96,500 \text{ C/mol}}{3.50 \text{ A}} \]
\[ t = 1.81 \times 10^5 \text{ s} \]
\[ t = 5.02 \text{ h} \]
1. Calculate the minimum time, in seconds, required to deposit 40.0 g of copper at the cathode of an electrolysis cell containing CuSO₄(aq) using a current of 20 000 mA.

2. How many minutes does it take to plate 0.925 g of silver onto the cathode of an electrolytic cell using a current of 1.55 A?

3. The nickel anode in an electrolytic cell decreases in mass by 1.20 g in 35.5 min. The oxidation half-reaction converts nickel atoms to nickel (III) ions. What is the constant current?

4. The following two half-reactions take place in an electrolytic cell with an iron anode and a chromium cathode.

\[ \text{Fe(s)} \rightarrow \text{Fe}^{2+} + 2e^- \]
\[ \text{Cr}^{3+} + 3e^- \rightarrow \text{Cr(s)} \]

During the process, the mass of the iron anode decreases by 1.75 g. Find the change in mass of the chromium cathode.

5. A student wishes to set up an electrolytic cell to plate copper onto a belt buckle. Predict the length of time it will take to plate out 2.5 g of copper from a copper (II) nitrate solution using a 2.5 A current. At which electrode should the belt buckle be attached?

6. Determine the mass of chlorine produced when a 200 A current flows for 24.0 h through a cell containing molten sodium chloride. At which electrode is the chlorine produced?

7. A trophy company is setting up a nickel plating cell using an electrolyte containing nickel (III) ions. Predict the current required to produce metal at a rate of 5.00 g/min.

8. In the electrolysis of a molten group II metal chloride 2.50 A of constant current is passed through a cell for 1.28 hours. Use the information provided below to determine the identity of the group II metal.

| Mass of cathode before application of current: | 25.720 g |
| Mass of cathode after application of current:  | 30.949 g |